

Name: _____

Key

Class: _____

1.4 and 1.5 Practice with Composition, Inverses, and Transformations

Find the following compositions if $f(x) = 2x^2 + 3$ and $g(x) = 4x^3 + 1$

1. $f(g(x))$

$$2(4x^3 + 1)^2 + 3$$

$$2(16x^6 + 8x^3 + 1) + 3$$

$$32x^6 + 16x^3 + 2 + 3$$

$$\boxed{32x^6 + 16x^3 + 5}$$

2. $g(f(x))$

$$4(2x^2 + 3)^3 + 1$$

$$4(4x^4 + 12x^2 + 9)(2x^2 + 3) + 1$$

$$4(8x^6 + 12x^4 + 24x^4 + 36x^2 + 18x^2 + 27) + 1$$

$$32x^6 + 48x^4 + 96x^4 + 144x^2 + 72x^2 + 108 + 1$$

$$\boxed{32x^6 + 144x^4 + 216x^2 + 109}$$

3. $f(g(-1))$

$$32(-1)^6 + 16(-1)^3 + 5$$

$$32 - 16 + 5$$

$$\boxed{21}$$

Find the following compositions if $f(x) = \frac{x-3}{x+2}$ and $g(x) = \frac{x}{x-1}$

4. $f(g(x))$

$$\frac{\frac{x}{x-1} - 3}{\frac{x}{x-1} + 2} \rightarrow \frac{\frac{x-3x+3}{x-1}}{\frac{x+2x-2}{x-1}}$$

$$\frac{-2x+3}{x-1} \cdot \frac{x-1}{3x-2}$$

$$\boxed{\frac{-2x+3}{3x-2}}$$

5. $g(f(x))$

$$\frac{\frac{x-3}{x+2}}{\frac{x-3}{x+2} - 1} \rightarrow \frac{\frac{x-3}{x+2}}{\frac{x-3-x-2}{x+2}}$$

$$\frac{x-3}{x+2} \cdot \frac{x+2}{-5}$$

$$\boxed{\frac{x-3}{-5}}$$

6. $g(f(5))$

$$\frac{5-3}{-5}$$

$$\boxed{-\frac{2}{5}}$$

Decomposition: $h(x) = f(g(x))$ and $h(x) = \frac{\sqrt{4x-1}}{3}$

7. Possible $f(x) = \frac{\sqrt{x}}{3}$

Possible $g(x) = 4x - 1$

8. Possible $f(x) = \frac{\sqrt{x-1}}{3}$

Possible $g(x) = 4x$

Find the inverse of the following functions.

9. $f(x) = \sqrt{\frac{x+1}{3}}$

$$x = \sqrt{\frac{y+1}{3}}$$

$$x^2 = \frac{y+1}{3}$$

$$3x^2 = y+1$$

$$\boxed{y^{-1} = 3x^2 - 1}$$

10. $h(x) = \frac{5x-1}{2x+4}$

$$y = \frac{5y-1}{2y+4} \rightarrow x(2y+4) = 5y-1$$

$$2xy + 4x = 5y - 1$$

$$2xy - 5y = -4x - 1$$

$$y(2x-5) = -4x-1$$

$$\boxed{y^{-1} = \frac{-4x-1}{2x-5}}$$

Confirm that the following functions are inverses (use composition). $f(x) = \frac{x+3}{x-4}$ and $g(x) = \frac{4x+3}{x-1}$

$$f(g(x)) = \frac{\frac{4x+3}{x-1} + 3}{\frac{4x+3}{x-1} - 4}$$

$$g(f(x)) = \frac{4\left(\frac{x+3}{x-4}\right) + 3}{\frac{x+3}{x-4} - 1}$$

$$\frac{4x+3+3x-3}{x-1} = \frac{7x}{x-1}$$

$$\frac{4x+3-4x+12}{x-1} = \frac{15}{x-1}$$

$$\frac{7x}{x-1} \div \frac{15}{x-1} = \frac{7x}{15} \neq x$$

$$\frac{4x+12+3x-12}{x-4} = \frac{7x}{x-4}$$

$$\frac{x+3-x+4}{x-4} = \frac{7}{x-4}$$

$$\frac{7x}{x-4} \div \frac{7}{x-4} = \frac{7x}{7} = x$$

Find the following if $f(x) = \frac{6x+1}{x-2}$, and $g(x) = \sqrt{2x-5}$

12. $f^{-1}(3)$

$$x = \frac{6y+1}{y-2}$$

$$y^{-1}(3) = \frac{7}{-3}$$

$$xy - 2x = 6y + 1$$

$$xy - 6y = 2x + 1$$

$$y(x-6) = 2x+1$$

$$y^{-1} = \frac{2x+1}{x-6}$$

$$\boxed{\frac{-7}{3}}$$

13. $g^{-1}(-1)$

$$x = \sqrt{2y-5}$$

$$y^{-1}(-1) = \frac{6}{2}$$

$$x^2 = 2y - 5$$

$$x^2 + 5 = 2y$$

$$y^{-1} = \frac{x^2 + 5}{2}$$

$$\boxed{3}$$

Determine the transformation of each function from its parent function.

14. $a(x) = 3e^{x-4}$

V stretch 3,
Right 4

15. $b(x) = \frac{-3}{x+1} - 2$

Ref. x axis, V stretch 3,
Left 1, Down 2

16. $c(x) = \sqrt[3]{-5x} + 12$

Ref. y axis, H. Shrink $\frac{1}{5}$,
up 12

17. $d(x) = \frac{1}{4}\sin(-x) + 1$

V. Shrink $\frac{1}{4}$, up 1,
Ref. y axis

18. $e(x) = \left|-\frac{x}{2}\right| - 1$

Ref. y axis, H. Stretch 2,
Down 1

19. $f(x) = -2\log(x-6) + 4$

Ref. x axis, V. Stretch 2,
Right 6, Up 4

Reflect the functions over the x and y axis

20. $g(x) = -4x^5 + 5x^4 + 3x^3 - 2x^2 + 10x - 1$

x axis: $4x^5 - 5x^4 - 3x^3 + 2x^2 - 10x + 1$

y axis: $4x^5 + 5x^4 - 3x^3 - 2x^2 - 10x - 1$

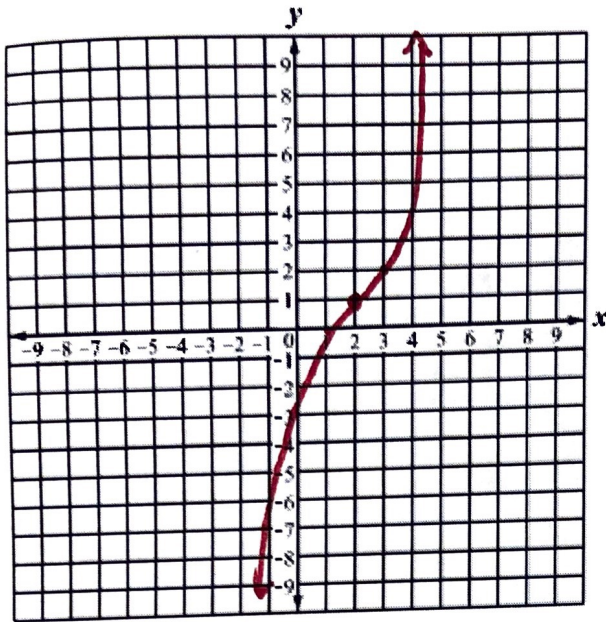
21. $h(x) = \frac{-5x^2+4x-2}{5x^2-4x+2}$

x axis: $\frac{5x^2-4x+2}{-4x+1}$ or $\frac{-5x^2+4x-2}{4x-1}$

y axis: $\frac{-5x^2-4x-2}{4x+1}$

Draw each function with its proper transformations. Then analyze the function below.

22. $j(x) = (x - 2)^3 + 1$



Domain: $(-\infty, \infty)$

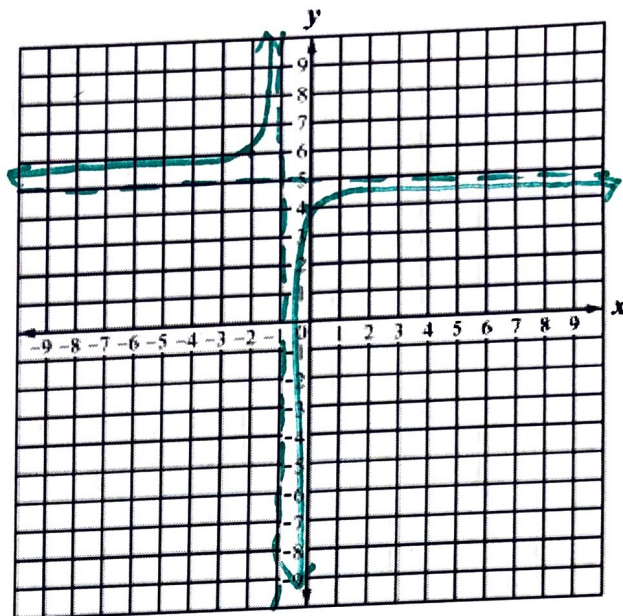
Range: $(-\infty, \infty)$

Continuity: continuous

Symmetry: none

End Behavior: $\lim_{x \rightarrow -\infty} j(x) = -\infty$ $\lim_{x \rightarrow \infty} j(x) = \infty$

23. $k(x) = -\frac{1}{x+1} + 5$



Domain: $(-\infty, -1) \cup (-1, \infty)$

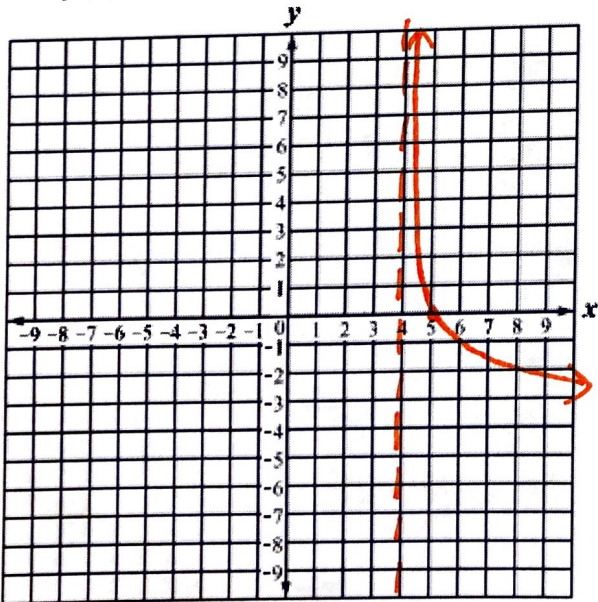
Range: $(-\infty, 5) \cup (5, \infty)$

Continuity: Infinite Discontinuity

Asymptotes: $x = -1, y = 5$

End Behavior: $\lim_{x \rightarrow -\infty} k(x) = 5$ $\lim_{x \rightarrow \infty} k(x) = 5$

24. $p(x) = -\log(x - 4)$

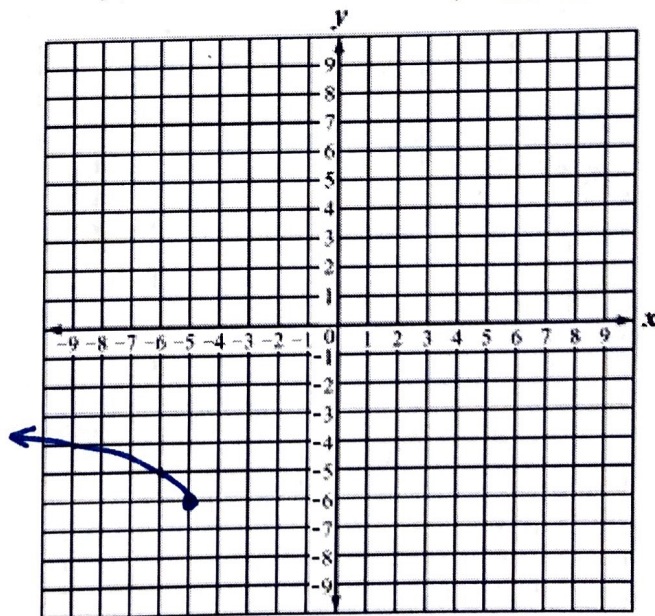


Domain: $(4, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = 4$

25. $q(x) = \sqrt{-x - 5} - 6$ $\sqrt{-(x+5)} - 6$



Domain: $(-\infty, -5]$

Range: $[-6, \infty)$

Extrema: Abs Min (-5, -6)